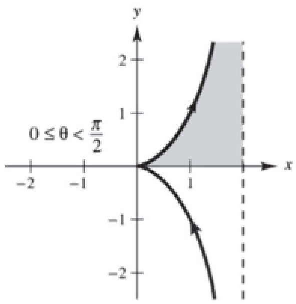
Homework5

1. Find the area of the region.(Use the result of Exercise 77.)

$$x = 2\ sin^2\theta$$
$$y = 2\ sin^2\theta\ tan\theta$$
$$0 \le \theta < \frac{\pi}{2}$$



2. Use the series representation of the function f to find $\lim_{x\to 0} f(x)$, if it exists.

$$f(x) = \frac{e^x - 1}{x}$$

3. Convert the polar equation to rectangular form and sketch its graph.

$$r = sec\ \theta\ tan\ \theta$$

Sol :

1.

$$\frac{dx}{d\theta} = 4\ sin\ \theta\ cos\ \theta$$

$$\begin{aligned} A &= \int_0^{\frac{\pi}{2}} 2sin^2\ \theta tan\ \theta(4sin\ \theta cos\ \theta)d\theta \\ &= 8\int_0^{\frac{\pi}{2}} sin^4\ \theta d\theta \\ &= 8\left[\frac{-sin^3\ \theta cos\ \theta}{4} - \frac{3}{8}sin\ \theta cos\ \theta + \frac{3}{8}\theta\right]_0^{\frac{\pi}{2}} = \frac{3\pi}{2} \end{aligned}$$

2.

$$\begin{aligned}
&Because \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \\
&e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \ \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!} \\
&and \quad \frac{e^x - 1}{x} = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \cdots \ \sum_{n=0}^{\infty} \frac{x^n}{(n+1)!} \\
&you\ have\ \lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \sum \frac{x^n}{(n+1)!} = 1
\end{aligned}$$

3.

$$\begin{aligned}
r &= sec\ \theta\ tan\ \theta \\
rcos\ \theta &= tan\ \theta \\
x &= \frac{y}{x} \\
y &= x^2
\end{aligned}$$